

A Simulation-based Inexact Two-stage Chance Constraint Quadratic Programming for Sustainable Water Quality Management under Dual Uncertainties

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Abstract

Water quality planning is complicated itself but further challenged by the existence of uncertainties and nonlinearities in terms of model formulation and solution. In this study, a simulation-based inexact two-stage chance constraint quadratic programming (SITCQP) approach was developed for water quality management. The SITCQP model was a hybrid of the two-stage stochastic programming (TSP), the chance constraint programming (CCP), the inexact quadratic programming (IQP) and the multi-segment stream water quality simulation. A water quality simulation model was provided for reflecting the relationship between the pollution-control actions before waste water discharge and the environmental responses after the discharge. The interval quadratic polynomials were employed to reflect the uncertainties and nonlinearities associated with the costs for wastewater treatment. Uncertainties derived from water quality standards were characterized as random variables with normal distributions. The proposed approach was applied to a hypothetical case in water quality management. Solutions from the SITCQP approach model were presented as combinations of deterministic, interval and distributional information, which could facilitate predictions for different forms of uncertainties. The results were valuable for helping decision makers generate alternatives between wastewater treatment and stream water quality management.

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Introduction

Water is one of the most important strategic resources for the development of economy and society. Water quality management is an essential task for preserving valuable water resources and facilitating sustainable socio-economic development in watershed systems (Van and Argiropoulos 2004; Kotti et al. 2005). Recently, due to limited water resources, it becomes challenging to alleviate the conflict between supply and demand of water and to reduce the pollutions caused by the water utilization. Furthermore, with the continuous economic development, water pollution is significantly increasing, which has become a major obstacle to sustainable development of water resources in watershed systems. Fortunately, a number of effective methods have been developed to examine the temporal and spatial impacts of alternative pollution-control actions in economic, environmental and ecological perspectives, and thus support decision makers in formulating and adopting cost-effective water quality management plans and policies (Wen and Lee 1998; Zou and Lung 2004; Kerachinan and Karamouz 2007; Perlman and Ostfeld 2008; Alp and Melching 2009; Sechi and Sulis 2009; Franceschini and Tsai 2010; Reichold et al. 2010; Zhang et al. 2011).

However, water quality management is usually affected by a variety of uncertainties raising from the stochasticity in hydrodynamic conditions and meteorological processes, the variability in the pollutant transport, the transmission time, the physicochemical processes, the dynamic interactions between pollutant loadings and receiving water bodies, the indeterminacy of available water and treated wastewater, etc (Babaeyan-Koopaei et al. 2003; Revelli and Ridolfi 2004; Huang and Chang 2003; Li and Huang 2009). Moreover, nonlinear relationship may also exist in different sectors of a water quality management system. For example, the system costs for water quality management are usually represented by a number of nonlinear functions. Nonlinear costs of the wastewater treatment plants, which are usually used as the objective in a water quality management model, are functions of wastewater discharge levels. Over the past decades, various methods dealing with uncertainties and nonlinearities have been developed for planning water quality management systems. Most of these methods can be grouped into stochastic mathematical programming (SMP), fuzzy mathematical programming (FMP), and/or interval mathematical programming (IMP) (Morgan et al. 1993; Wu et al. 1997; Chen and Chang

1998; Chang et al. 2001; Mujumdar and Saxena 2004; Maqsood et al. 2005; Lee and Chang 2005; Karmakar and Mujumdar 2007; Cunha et al. 2009; Xu and Qin 2010; Ray et al. 2010).

The two-stage stochastic programming (TSP) method is a typical SMP method, which is an effective alternative for the problems where an analysis of policy scenarios is desired and uncertainties are represented by known probability distributions. In the TSP, an initial decision is made based on some uncertain future events. When these future uncertainties are later resolved, a recourse or corrective action is correspondingly taken. The initial decision is called the first-stage decision, and the corrective action is called the second-stage decision. Therefore, the TSP can provide an effective link between policies and associated economic penalties caused by improper policies (Seifi and Hipel 2001; Li and Huang 2007). The TSP methods have been widely explored over the past decades (Ruszczynski and Swietanowski 1997; Huang and Loucks 2000; Luo et al. 2006; Li and Huang 2007). For example, Maqsood et al. (2005) developed an interval-parameter fuzzy two-stage stochastic programming approach for water resources management; Li and Huang (2009) proposed an inexact two-stage stochastic quadratic programming approach with application to water quality management. In general, the TSP methods can handle uncertainties expressed as probability density functions (PDFs) as well as economic penalties with recourse against any infeasibility, which requires clear definitions for probability distributions. However, it is of difficulty to obtain such definitions because the quality of the obtained information is usually inefficient in many practical problems. Moreover, the TSP has significant drawback in dealing with the risk of violating uncertain system constraints.

The chance-constrained programming (CCP) method can be a promising solution for addressing the above challenges. The CCP model can effectively reflect the reliability of satisfying (or risk of violating) system constraints under uncertainty. In fact, the CCP method does not require that all constraints should be totally satisfied. Based on the given probabilities of constraint violations, the CCP can be used to convert a stochastic programming model into an equivalent model with deterministic settings, leading to reduction of system complexity. This model can be further incorporated with other uncertain optimization methods within a general framework. There have been many applications of the CCP method to environmental management problems (Charnes et

al. 1972; Charnes and Cooper 1983; Morgan et al. 1993; Liu et al. 2003; Yang and Wen 2005; Karmakar and Mujumdar 2007; Qin and Huang 2009; Xu and Qin 2010). For example, Huang (1998) developed an inexact chance-constrained programming approach for water quality management based on the introduction of allowing probability distributions and discrete intervals into the optimization process. More recently, Zhang et al. (2009) proposed a robust chance-constrained fuzzy possibilistic programming model for water quality management within an agricultural system; Xie et al. (2011) developed an inexact-chance-constrained water quality management (ICC-WQM) model and applied it to a case study of chemical-industry planning in Binhai New Area of Tianjin, China. Although the CCP is effective in reflecting probability distributions of the constraints' right-hand sides (b_i), it lacks of ability in handling uncertainties of the left-hand side coefficients in constraints (a_i) or coefficients in the objective functions (c_j) (taking a simple model for example: $\min f = C^T x$, subject to $Ax \leq b_i(t)$). More importantly, this method is unable to handle such difficulties when nonlinearities exist in the objective function.

The inexact quadratic programming (IQP) method is effective in not only reflecting nonlinearities in the objective function, but also incorporating uncertainties in the quadratic programming optimization process. The IQP model can be transformed into a number of deterministic sub-models to generate interval solutions that are feasible and stable in the given decision space (Chen and Huang 2001). These sub-models can easily be solved by quadratic programming packages provided by many programming software (e.g. LINGO®, MATLAB®), and the global optimums can be obtained if the Kuhn–Tucker conditions are satisfied (Hu et al. 2012). However, the IQP method encountered difficulties in reflecting uncertainties expressed as random variables, and it was incapable of examining economic consequences of violating some overriding policies that were considered beyond the scope of the planning cycle.

Based on the above analyses, an inexact two-stage chance constraint quadratic programming (ITCQP) method is developed for water quality management, which is a hybrid of the TSP, the CCP and the IQP. Therefore, the developed ITCQP will be capable of handling uncertainties expressed as probability distributions and interval values, and dealing with nonlinearities in the objective function. Furthermore, in order to address the challenge in data unavailability for the

conventional optimization method, a multi-segment stream water quality simulation model based on the O'Connor model (O'Connor and Dobbins 1958) will be coupled into the ITCQP model. Biochemical oxygen demand (BOD) and dissolved oxygen (DO) are two of the important indicators for water quality. In 1925, Streeter and Phelps derived the classic equations for simulating DO and BOD in streams (Cox 2003). These equations have formed the basis of many water quality models that have been widely introduced and applied in the world. The multi-segment O'Connor model is able to predict stream water quality responses under various wastewater discharge scenarios. Therefore, for a typical water quality planning problem, the developed simulation-based ITCQP (i.e. SITCQP) method will provide an effective link between environmental requirements and cost expressed as penalties or opportunity losses. A case study will then be provided for demonstrating the applicability of the developed methodology. The results can help decision makers to manage stream water quality and gain insight into the tradeoffs between the environmental benefits and treatment costs.

Methodology

General Framework

The multiple point source wastewater discharge problems are described as meeting both emissions standards and water quality standards at the lowest system cost or highest system profit in the study (Loucks et al. 1981; Luo et al. 2006; Qin et al. 2009). The developed SITCQP model can effectively deal with these problems. In the model, the cost function is expressed in an interval quadratic format, and the water quality constraints derive from a water quality simulation model. The detailed procedures is as follows: (1) applying a multi-segment water quality simulation model to predict stream water quality responses under various wastewater discharge scenarios which will be employed as constraints in the final model; (2) utilizing the economic intervals to formulate the cost function of wastewater treatment, which is a nonlinear function and embedded in the objective function in an interval quadratic format; (3) determining the probability distributions of water quality standards and identifying the allowable risk levels of violating the water quality constraints; (4) establishing the SITCQP model and converting it to an equivalent IQP model; (5) obtaining the optimal interval solutions based on the IQP solution

algorithms. The details about the components of the SITCQP model will be described in the following sections.

Multi-segment Stream Water Quality Simulation

Water quality simulation models have become useful tools for supporting environmental management in the past decades, such as the Streeter–Phelps model, O’Conner model, Dobbins model, and Thomas model (Rauch et al. 1998). In the water quality simulation, segmenting a stream into multiple reaches is necessary because a series of wastewater outlets scatter along the stream with temporally and spatially variational loadings (Grigg 1985; Murty et al. 2006; Qin et al. 2009). Water quality at each section is affected by various sources from the upper stream. In this study, the basic BOD–DO relations are generated based on the O’Connor model.

Accordingly, the BOD load and DO deficit in relation to the m wastewater-discharge sources can be predicted as follows (O’Connor and Dobbins 1958; Thomann and Mueller 1987; Eckenfelder 2000; Karmakar and Mujumdar 2007; Li et al. 2007; Xie et al. 2011):

$$L_{cn} = \prod_{j=1}^n L_{c0} e^{-(k_d+k_s)x_j/u_{x,j}} + \prod_{j=2}^n (1-\eta_1) e^{-(k_d+k_s)x_j/u_{x,j}} BOD_{c,1} + \cdots \\ + (1-\eta_{m-1}) e^{-(k_d+k_s)x_n/u_{x,n}} BOD_{c,m-1} + (1-\eta_m) BOD_{c,m} \quad (1a)$$

$$D_n = D_{n-1} e^{-k_a x/u_x} + \frac{k_d L_{c,n-1}}{k_a - (k_d + k_s)} \left[e^{-(k_d+k_s)x/u_x} - e^{-k_a x/u_x} \right] \quad (1b)$$

where j is defined as a segmentation of the stream between source j to $j+1$ ($j = 1, 2, \dots, n$); m is the number of wastewater discharge sources; L_{c0} is the initial BOD in the stream immediately after discharge (mg/L); k_d is the CBOD decay rate in the stream, $[T^{-1}]$; k_s is the BOD decay rate due to sedimentation, $[T^{-1}]$; $L_{c,n-1}$ and L_{cn} are the respective BOD loads in the stream at the beginnings of reaches $n-1$ and n ; i denotes wastewater discharge source, and $i = 1, 2, \dots, m$; η_i is the wastewater-treatment efficiency at discharge source i ; BOD_{ci} is the total amount of BOD to be disposed of at source i (kg/day); D is the oxygen deficit concentration, $[ML^{-3}]$; k_a is the reaeration rate, $[ML^{-3}]$; u_x is the average stream flow rate, $[L/T]$; and x is the flow longitudinal

distance along the x axis, [L].

Cost function for Wastewater Treatment

Uncertainties may exist in a number of modeling parameters due to human-induced and/or natural variability (Vicens et al. 1975; Li et al. 2007). For example, the volume and strength of industrial waste-water can be defined in terms of units of production (e.g., gallons of wastewater per ton of pulp produced), and their variations can be estimated through identifying a statistical distribution for each source (Eckenfelder 2000). The magnitudes of the variations depend on the diversity of manufactured products, the process of operations contributing to the wastewater flow, and the mode of production (batch or continuous) (Eckenfelder 2000). When uncertainties expressed as interval values, the cost function for wastewater treatment can be expressed as the following inexact form (Loucks et al. 1981; Haith 1982; Grigg 1985; Li and Huang 2009; Jan et al. 2011):

$$C^{\pm} = k_1^{\pm} Q^{\pm k_2} + k_3^{\pm} Q^{\pm k_4} \quad (2)$$

where C is the treatment cost for wastewater under a specific technology, including costs for construction, operation and maintenance (\$10⁴/yr); superscripts '+' and '-' represent lower and upper bounds of the interval parameters, respectively; Q is the average wastewater flow (10³ m³/day), and k_1 and k_3 are the cost-function coefficients ($k_1, k_3 > 0$), while k_2 and k_4 are the economies-of-scale indexes of the capital and treatment costs ($0 < k_2, k_4 < 1$). Different wastewater-treatment processes (with different efficiencies) may lead to varied levels for k_2 and k_4 . In China, the cost functions for wastewater treatment plants have been extensively investigated, and most of which are formulated based on the treatment efficiencies of the biological oxygen demand. Fig. 1 shows a typical inexact nonlinear function between wastewater flow vs treatment cost while the removal efficiency of wastewater flow keeps constant. The relationship reflects the economy-of- capacity where the treatment cost of pollutant loading is changing with the wastewater capacity (Thuesen et al. 1977). It is indicated that the relationships could be approximated by a quadratic function, such as $Cost = aQ + bQ^2$, where a and b are cost coefficients, with a reasonable degrees of errors (Huang et al. 1995). When the efficiency of

wastewater treatment changes, the economy-of-scale effect can be reflected in the cost function by a nonlinear relationship expressed in a power form.

SITCQP Model Formulation

Accordingly, a simulation-based inexact two-stage chance constraint quadratic programming (SITCQP) method can be developed. The developed model can deal with the uncertainties in the both the left-hand and right-hand sides of the constraints as well as the objective-function coefficients presented as random variables with known probability distributions. Furthermore, it can also provide information on the trade-offs among the objective function value, tolerance values of the constraint, and the prescribed level of probability. The objective function is to maximize the expected value of net system benefit under various constraints. The constraints include the BOD-loading allowance for each discharge resource as well as the allowable BOD and DO-deficit levels in each stream segment.

$$\begin{aligned} \max f^{\pm} = & l \sum_{i=1}^I \sum_{k=1}^K NB_{ik}^{\pm} T_{ik}^{\pm} + m \sum_{i=1}^I \sum_{k=1}^K \sum_{h=1}^S p_{ikh} NB_{ik}^{\pm} (X_{ikh}^{\pm} - T_{ik}^{\pm}) \\ & - n \sum_{i=1}^I \sum_{k=1}^K \sum_{h=1}^S p_{ikh} \left[c_{ikh}^{\pm} w_{ikh}^{\pm} X_{ikh}^{\pm} + d_{ikh}^{\pm} (w_{ikh}^{\pm} X_{ikh}^{\pm})^2 \right] \end{aligned} \quad (3a)$$

subject to

(1) BOD discharge constraints:

$$(1 - \eta_{ik}^{\pm}) X_{ikh}^{\pm} w_{ikh}^{\pm} C_{ik}^{\pm} \leq S_{ik}^{\pm}, \quad \forall i, k \quad (3b)$$

(2) Chance constraints:

Maximum allowable BOD discharge and DO-deficit constraints:

$$\begin{cases} \Pr(L_j^{\pm} \leq R_{jk, BOD}^{\pm}) \geq 1 - q \\ \Pr(D_j^{\pm} \leq R_{jk, D}^{\pm}) \geq 1 - q \end{cases}, \quad \forall j, k \quad (3c)$$

$$\begin{cases} L_j^{\pm} \leq R_{jk, BOD}^q \\ D_j^{\pm} \leq R_{jk, D}^q \end{cases}, \quad \forall j, k \quad (3d)$$

(3) Product demand constraints:

$$T_{ik, \min} \leq T_{ik}^{\pm} \leq T_{ik, \max}, \quad \forall i, k \quad (3e)$$

(4) Non-negative constraints:

$$X_{ikk}^{\pm} \geq 0, \forall i, k, h \quad (3f)$$

where the superscripts '+' and '-' represent the lower and upper bounds of the interval parameters, respectively; j is the index of reach; i denotes the name of wastewater discharge source; l, m and n are the coefficients of the objective function; f^{\pm} is the net system benefit over the planning horizon (\$); p_{ik} denotes probability of random wastewater discharge rate at source i in period k with level h (%); h denotes the level of wastewater discharge rate at each source; k is the index for planning period; NB_{ik}^{\pm} is the net benefit per unit product from source i (\$/unit product); C_{ik}^{\pm} is the BOD concentration of raw wastewater generated at source i in period k (kg/m³); c_{ik}^{\pm} and d_{ik}^{\pm} are the coefficients of treatment cost function for source i during period k ($c_{ik}^{\pm}, d_{ik}^{\pm} > 0$); $R_{jk,BOD}^{\pm}$ is the designated BOD concentration at the beginning of reach j (mg/L); $R_{jk,D}^{\pm}$ is the allowable DO deficit at the end of reach j (mg/L); S_{ik}^{\pm} is the BOD discharge allowance for source i during period k (tonne/day); $T_{ik,min}$ and $T_{ik,max}$ are the minimum and maximum demands for product i during period k (unit/day), respectively; T_{ik}^{\pm} is product target pre-regulated by source i during period k (the first-stage decision variable) (unit/day); w_{ikk}^{\pm} is the random wastewater discharge rate at source i in period k with level h (m³/unit product); X_{ikk}^{\pm} means the production level of source i during period k with level h , which is affected by the random BOD generation rates and the environmental requirements (the second-stage decision variable) (unit/day).

Solution methods

Generally, the SITCQP is a simulation-based inexact two-stage chance constraint quadratic programming, and the form of which is as follows:

$$\max f^{\pm} = \sum_{j=1}^{n_1} \left[c_j^{\pm} x_j^{\pm} + d_j^{\pm} (x_j^{\pm})^2 \right] - \sum_{j=1}^{n_2} \sum_{k=1}^v p_k \left[e_j^{\pm} y_{jk}^{\pm} + g_j^{\pm} (y_{jk}^{\pm})^2 \right] \quad (4a)$$

subject to

$$\sum_{j=1}^{n_1} a_{vj}^{\pm} x_j^{\pm} \leq b_v^{\pm}, v = 1, 2, \dots, m_1 \quad (4b)$$

$$\sum_{j=1}^{n_1} a_{vj}^{\pm} x_j^{\pm} + \sum_{j=1}^{n_2} a_{vj}^{t\pm} y_{jk}^{\pm} \geq \tilde{w}_k^{\pm}, t = 1, 2, \dots, m_2; k = 1, 2, \dots, v \quad (4c)$$

$$\sum_{j=1}^{n_1} a_{sj}^{\pm} x_j^{\pm} \leq B_s(t)^{(q_s)}, s = 1, 2, \dots, m_3 \quad (4d)$$

$$x_j^{\pm} \geq 0, j = 1, 2, \dots, n_1 \quad (4e)$$

$$y_{jk}^{\pm} \geq 0, j = 1, 2, \dots, n_1; k = 1, 2, \dots, v \quad (4f)$$

where c_j^{\pm} , d_j^{\pm} , e_j^{\pm} and g_j^{\pm} are interval parameters/variables; x_j^{\pm} represents the first-stage decision variables, which has to be decided before the actual realizations of the random variables; y_{jk}^{\pm} denotes the second-stage decision variables, which is related to the recourse actions against any infeasibilities arising due to particular realizations of the uncertainties; a_{vj}^{\pm} , a_{vj}^{\pm} , $a_{vj}^{t\pm}$, a_{sj}^{\pm} and b_v^{\pm} are the coefficients of constraints; \tilde{w}_k^{\pm} are random variables with probability levels p_k , where $k = 1, 2, \dots, v$ and $\sum p_k = 1$; $B_s(t)^{(q_s)}$ is the cumulative distribution function of q_s ; and q_s is the probability of violating constraint, $q_s \in [0, 1]$. The decision variables Z_j are introduced to identify an optimized set of the first-stage values for supporting the related policy analyses.

Let $x_j^{\pm} = x_j^- + \Delta x_j Z_j$, where $\Delta x_j = x_j^+ - x_j^-$ and $Z_j \in [0, 1]$. Thus, model (4) can be converted into:

$$\begin{aligned} \max f^{\pm} = & \sum_{j=1}^{n_1} \left[c_j^{\pm} (x_j^- + \Delta x_j Z_j) + d_j^{\pm} (x_j^- + \Delta x_j Z_j)^2 \right] - \\ & \sum_{j=1}^{n_2} \sum_{k=1}^v p_k \left[e_j^{\pm} y_{jk}^{\pm} + g_j^{\pm} (y_{jk}^{\pm})^2 \right] \end{aligned} \quad (5a)$$

subject to

$$\sum_{j=1}^{n_1} a_{vj}^{\pm} (x_j^- + \Delta x_j Z_j) \leq b_r^{\pm}, r = 1, 2, \dots, m_1 \quad (5b)$$

$$\sum_{j=1}^{n_1} a_{vj}^{\pm} (x_j^- + \Delta x_j Z_j) + \sum_{j=1}^{n_2} a'_{vj} y_{jh}^{\pm} \geq \tilde{w}_h^{\pm}, t = 1, 2, \dots, m_2; h = 1, 2, \dots, v \quad (5c)$$

$$\sum_{j=1}^{n_1} a_{vj}^{\pm} (x_j^- + \Delta x_j Z_j) \leq B_s(t)^{(q_s)}, s = 1, 2, \dots, m_3 \quad (5d)$$

$$x_j^- + \Delta x_j Z_j \geq 0, j = 1, 2, \dots, n_1 \quad (5e)$$

$$y_{jh}^{\pm} \geq 0, j = 1, 2, \dots, n_1; h = 1, 2, \dots, v \quad (5f)$$

$$0 \leq Z_j \leq 1, j = 1, 2, \dots, n_1 \quad (5g)$$

where Z_j , x_j^- and y_{jh}^{\pm} are decision variables. Then, when the coefficients (e_j^{\pm} and g_j^{\pm}) related to the second-stage decision variables (y_{jh}^{\pm}) in model (5) have the same sign, the SITCQP can be directly transformed into two deterministic sub-models based on an interactive algorithm (Chen and Huang 2001):

$$\begin{aligned} \max f^+ = & \sum_{j=1}^{n_1} \left[c_j^+ (x_j^- + \Delta x_j Z_j) + d_j^+ (x_j^- + \Delta x_j Z_j)^2 \right] \\ & - \sum_{j=1}^{k_2} \sum_{h=1}^v p_h \left[e_j^- y_{jh}^- + g_j^- (y_{jh}^-)^2 \right] - \sum_{j=k_2+1}^{n_2} \sum_{h=1}^v p_h \left[e_j^- y_{jh}^+ + g_j^- (y_{jh}^+)^2 \right] \end{aligned} \quad (6a)$$

subject to

$$\sum_{j=1}^{n_1} |a_{vj}|^+ \text{Sign}(a_{vj}^+) (x_j^- + \Delta x_j Z_j) \leq b_r^+, \forall r \quad (6b)$$

$$\begin{aligned} & \sum_{j=1}^{n_1} |a_{vj}|^+ \text{Sign}(a_{vj}^+) (x_j^- + \Delta x_j Z_j) + \sum_{j=1}^{n_1} |a'_{vj}|^+ \text{Sign}(a'_{vj}^+) y_{jh}^- \\ & + \sum_{j=k_2+1}^{n_2} |a'_{vj}|^- \text{Sign}(a'_{vj}^-) y_{jh}^+ \geq \tilde{w}_h^-, \forall t, h \end{aligned} \quad (6c)$$

$$\sum_{j=1}^{n_1} |a_{vj}|^+ \text{Sign}(a_{vj}^+) (x_j^- + \Delta x_j Z_j) \leq B_s(t)^{(q_s)}, s = 1, 2, \dots, m_3 \quad (6d)$$

$$x_j^- + \Delta x_j Z_j \geq 0, j = 1, 2, \dots, n_1 \quad (6e)$$

$$y_{jh}^- \geq 0, j = 1, 2, \dots, k_2; \forall h \quad (6f)$$

$$y_{jh}^+ \geq 0, j = k_2 + 1, k_2 + 2, \dots, n_2; \forall h \quad (6g)$$

$$0 \leq Z_j \leq 1, j = 1, 2, \dots, n_1 \quad (6h)$$

where Z_j, x_j^-, y_{jh}^- and y_{jh}^+ are decision variables; $y_{jh}^- (j = 1, 2, \dots, k_2)$ are decision variables with positive coefficients in the objective function; $y_{jh}^+ (j = k_2 + 1, k_2 + 2, \dots, n_2)$ are decision variables with negative coefficients. Solutions of $y_{jh}^-_{opt} (j = 1, 2, \dots, k_2)$, $y_{jh}^+_{opt} (j = k_2 + 1, k_2 + 2, \dots, n_2)$, and Z_{jopt} can be obtained through solving sub-model (6). The optimized first-stage variables are $x_{jopt}^\pm = x_j^\pm + \Delta x_j Z_{jopt} (j = 1, 2, \dots, n_1)$ then, the sub-model corresponding to the lower-bound objective-function value (f) is:

$$\begin{aligned} \max f^- = & \sum_{j=1}^{n_1} \left[c_j^- (x_j^- + \Delta x_j Z_{jopt}) + d_j^- (x_j^- + \Delta x_j Z_{jopt})^2 \right] \\ & - \sum_{j=1}^{k_2} \sum_{h=1}^v p_h \left[e_j^+ y_{jh}^+ + g_j^+ (y_{jh}^+)^2 \right] - \sum_{j=k_2+1}^{n_2} \sum_{h=1}^v p_h \left[e_j^- y_{jh}^- + g_j^- (y_{jh}^-)^2 \right] \end{aligned} \quad (7a)$$

subject to

$$\sum_{j=1}^{n_1} |a_{rj}| \text{Sign}(a_{rj}^-) (x_j^- + \Delta x_j Z_j) \leq b_r^-, \forall r \quad (7b)$$

$$\begin{aligned} & \sum_{j=1}^{n_1} |a_{tj}| \text{Sign}(a_{tj}^-) (x_j^- + \Delta x_j Z_j) + \sum_{j=1}^{n_1} |a_{tj}'| \text{Sign}(a_{tj}'^-) y_{jh}^+ \\ & + \sum_{j=k_2+1}^{n_2} |a_{tj}'| \text{Sign}(a_{tj}'^+) y_{jh}^- \geq \tilde{w}_t^+, \forall t, h \end{aligned} \quad (7c)$$

$$\sum_{j=1}^{n_1} |a_{sj}| \text{Sign}(a_{sj}^-) (x_j^- + \Delta x_j Z_j) \leq B_s(t)^{(q_s)}, s = 1, 2, \dots, m_3 \quad (7d)$$

$$y_{jh}^+ \geq y_{jhopt}^-, j = 1, 2, \dots, k_2; \forall h \quad (7e)$$

$$0 \leq y_{jh}^- \leq y_{jhopt}^+, j = k_2 + 1, k_2 + 2, \dots, n_2; \forall h \quad (7f)$$

where $x_j^-, y_{jh}^+ (j = 1, 2, \dots, k_2)$ and $y_{jh}^- (j = k_2 + 1, k_2 + 2, \dots, n_2)$ are decision variables. Solutions of $y_{jh}^+ (j = 1, 2, \dots, k_2)$ and $y_{jh}^- (j = k_2 + 1, k_2 + 2, \dots, n_2)$ can be obtained through solving sub-model (7). Thus, the solutions can be obtained as follows:

$$x_{j \text{ opt}}^\pm = x_j^- + \Delta x_j Z_{j \text{ opt}}, j = 1, 2, \dots, n_1 \quad (8a)$$

$$y_{jh \text{ opt}}^\pm = [y_{jh \text{ opt}}^-, y_{jh \text{ opt}}^+], j = 1, 2, \dots, n_1; \forall h \quad (8b)$$

$$f_{\text{opt}}^\pm = [f_{\text{opt}}^-, f_{\text{opt}}^+] \quad (8c)$$

However, when the coefficients e_j^\pm and g_j^\pm have different signs, it is difficult to determine whether y_{jh}^- or y_{jh}^+ corresponds to the desired f^+ . Based on the derivative algorithm proposed by Chen and Huang (2001), let all left-hand-side and/or right-hand-side coefficients be equal to their mid-values and, then, model (5) can be converted into a mid-value inexact two-stage chance constraint quadratic programming problem as follows:

$$\begin{aligned} \max f_{\text{mv}} = & \sum_{j=1}^{n_1} \left[c_{j \text{ mv}} (x_j^- + \Delta x_j Z_j) + d_{j \text{ mv}} (x_j^- + \Delta x_j Z_j)^2 \right] - \\ & \sum_{j=1}^{n_2} \sum_{h=1}^v p_h \left[e_{j \text{ mv}} y_{jh \text{ mv}} + g_{j \text{ mv}} (y_{jh \text{ mv}})^2 \right] \end{aligned} \quad (9a)$$

subject to

$$\sum_{j=1}^{n_1} a_{rj \text{ mv}} (x_j^- + \Delta x_j Z_j) \leq b_{r \text{ mv}}, r = 1, 2, \dots, m_1 \quad (9b)$$

$$\sum_{j=1}^{n_1} a_{tj \text{ mv}} (x_j^- + \Delta x_j Z_j) + \sum_{j=1}^{n_2} a'_{tjh \text{ mv}} y_{jh \text{ mv}} \geq \tilde{w}_{tjh \text{ mv}}, t = 1, 2, \dots, m_2; h = 1, 2, \dots, v \quad (9c)$$

$$\sum_{j=1}^{n_1} a_{sj \text{ mv}} (x_j^- + \Delta x_j Z_j) \leq B_s(t)^{(q_s)}, s = 1, 2, \dots, m_3 \quad (9d)$$

$$x_j^- + \Delta x_j Z_j \geq 0, j = 1, 2, \dots, n_1 \quad (9e)$$

$$y_{jh \text{ mv}} \geq 0, j = 1, 2, \dots, n_1; h = 1, 2, \dots, v \quad (9f)$$

$$0 \leq Z_j \leq 1, j = 1, 2, \dots, n_1 \quad (9g)$$

where $c_{j\text{mv}}, d_{j\text{mv}}, e_{j\text{mv}}, g_{j\text{mv}}, a_{rj\text{mv}}, a_{tj\text{mv}}, a_{tj\text{mv}}^t, a_{oj\text{mv}}$ and $b_{r\text{mv}}$ are mid-value of c_j^\pm , d_j^\pm , e_j^\pm , g_j^\pm , a_{rj}^\pm , a_{tj}^\pm , $a_{tj}^{\pm t}$, a_{oj}^\pm and b_r^\pm (e.g., $c_{j\text{mv}} = (c_j^- + c_j^+) / 2$). Then, the optimal solutions for $(y_{jk\text{mv}})_{\text{opt}}$ can be obtained by solving sub-model (9). According to Chen and Huang (2001), the bound distribution for y_{jk}^\pm can be identified according to the following criteria:

$$f^+(y_{jk\text{opt}}^+) \geq f^+(y_{jk\text{opt}}^-), \text{ when } 2g_j^+(y_{jk\text{mv}})_{\text{opt}} + e_j^+ > 0 \quad (10a)$$

$$f^+(y_{jk\text{opt}}^+) \leq f^+(y_{jk\text{opt}}^-), \text{ when } 2g_j^+(y_{jk\text{mv}})_{\text{opt}} + e_j^+ < 0 \quad (10b)$$

If criterion (10a) is satisfied, then y_{jk}^+ corresponds to f^+ ; if criterion (10b) holds, y_{jk}^- corresponds to f^+ . Accordingly, model (5) can be converted into two sub-models that correspond to f^+ and f^- , based on the previously described solution algorithm. Obviously, the derivative algorithm is useful for the applications of SITCQP to large-scale problems where many coefficients (corresponding to the second-stage variables) have different signs.

Case Study

Overview of the study system

In a regional system, water pollution problems are usually characterized by many sewage discharging sources at different scales, causing adverse impacts on receptors. Many factors like properties of pollutants, the technology of wastewater treatment, and the control measures that affect the water quality management system cannot be quantified with certainty. Since it is either economically infeasible or technically impossible to design process with zero wastewater discharge, water quality managers always intend to select alternative which can meet the national environmental standards and meanwhile maximize the economic benefits. Therefore, the

problem under consideration is the identification of cost-effective pollution-control actions, so as to maximize the interests of regional industry and protect water from being polluted by industrial wastewater under uncertain environmental, economic and disposal conditions.

In this study, a hypothetical water quality management system, which includes six outfalls scatter along the stream, is provided to demonstrate the applicability of the proposed approach. Fig.2 shows the schematic diagram of the study system. The stream with a tributary is segmented into nine reaches which are marked from Y_1 to Y_9 , with reach Y_1 being at the upstream end, reach Y_9 at the downstream end and reaches $Y_3 \sim Y_5$ along the tributary. The discharged wastewater enters the stream from the municipal outlet (i.e. wastewater treatment plant) located in reach Y_2 , the industry outlets in reach Y_3 , Y_4 , Y_6 and Y_7 (i.e. a paper mill, a chemical plant, a tannery plant and a tobacco factory), and the recreational sector outlet in reach Y_9 . Details of the related stream parameters are provided in Fig.2. The planning horizon of the study problem is 15 years, which is further divided into three periods with 5 years intervals (i.e. 1825 days). The objective is to maximize the expected net benefit subject to the environmental requirements under uncertainty over the planning horizon. Policies in terms of the related industrial activities and the wastewater discharges are critical for ensuring maximized system benefit and safe water quality.

Table 1 provides the wastewater-discharge rates and the associated probability levels. Due to the regional economic development and population growth, the wastewater discharges keep increasing in the planning horizon. Moreover, significant variations in wastewater discharges exist among different sectors. To guarantee the stream water quality, effective wastewater treatment measures have to be adopted at point sources such as industrial sites. Table 2 presents the treatment efficiencies as well as the raw BOD concentrations at different discharge sources. Table 3 shows the related economic data. The costs for wastewater treatment are approximated as inexact nonlinear functions of wastewater flows, and they monotonically rise as the wastewater flow increase (Thuesen et al. 1977; Haith 1982; Li and Huang 2009). Table 4 presents the allowable BOD-loading level for each pollution source as regulated by the local authority (SEPA 2002). According to the environmental standards for surface water quality in China (SEPA 2002), the water bodies with BOD level of 4 mg/L and the DO level of 6 mg/L are defined as Class-III water (residential and fishery); the standards for BOD and DO in Class IV water (industrial and

recreational) are 6 and 5 mg/L respectively. For the local environmental agencies, the strictest national surface-water standards for BOD and DO are 3 and 6 mg/L (Class II, for highly protected areas such as spawning grounds and habitats of precious aquatic lives) respectively; the most dispersive ones are 6 and 3 mg/L (Class IV), respectively; and the intermediate ones are 4 and 5 mg/L (Class III) respectively (SEPA 1996, 2002). Based on the above standards, it is assumed that the water environmental standards (right-hand sides of constraints in the SITCQP model) are random variables following normal distributions where the mean values of BOD and DO are 4 and 5 mg/L, respectively. In addition, the standard deviations for both parameters are set to 0.5. Table 5 provides the minimum and maximum market demands for each economic activity. Table 6 presents the environmental standard constraints in different q_i .

SITCQP model for water quality management

Generally, the complexities of the water quality management problem in this case include: (a) multiple outlets exist in the study area; (b) many parameters are uncertain and are available as probabilistic distributions and/or discrete intervals; (c) the randomness of some parameters can affect the model results and the decision outcomes; (d) dynamic interactions exist between pollutant loading and water quality; (e) nonlinearities exist in the waste treatment cost functions. The proposed SITCQP model is considered to be suitable for tackling such uncertainties and complexities. Accordingly, the proposed model can be formulated as follows:

$$\begin{aligned} \max f^{\pm} = & 1825 \sum_{i=1}^5 \sum_{k=1}^3 NB_{ik}^{\pm} T_{ik}^{\pm} + 5 \sum_{k=1}^3 NB_{6k}^{\pm} T_{6k}^{\pm} \\ & + 1825 \sum_{i=1}^5 \sum_{k=1}^3 \sum_{l=1}^s p_{ikl} NB_{ik}^{\pm} (X_{ikl}^{\pm} - T_{ik}^{\pm}) \\ & + 5 \sum_{k=1}^3 \sum_{l=1}^s p_{6kl} NB_{6k}^{\pm} (X_{6kl}^{\pm} - T_{6k}^{\pm}) \\ & - 5 \sum_{i=1}^5 \sum_{k=1}^3 \sum_{l=1}^s p_{ikl} \left[c_{ikl}^{\pm} w_{ikl}^{\pm} X_{ikl}^{\pm} + d_{ikl}^{\pm} (w_{ikl}^{\pm} X_{ikl}^{\pm})^2 \right] \end{aligned} \quad (11a)$$

subject to

(1) BOD discharge constraints:

$$(1 - \eta_{ik}^{\pm}) X_{ikl}^{\pm} w_{ikl}^{\pm} C_{ik}^{\pm} \leq S_{ik}^{\pm}, \quad \forall i, k \quad (11b)$$

(2) Chance constraints:

Maximum allowable BOD discharge and DO-deficit constraints:

$$\begin{cases} \Pr(L_j^+ \leq R_{jk,BOD}^+) \geq 1-q \\ \Pr(D_j^+ \leq R_{jk,D}^+) \geq 1-q \end{cases}, \forall j, k \quad (11c)$$

$$\begin{cases} L_j^+ \leq R_{jk,BOD}^q \\ D_j^+ \leq R_{jk,D}^q \end{cases}, \forall j, k \quad (11d)$$

Maximum allowable BOD discharge:

The tributary:

$$1.7134 + (1 - \eta_{2k}^+) X_{2kk}^+ w_{2kk}^+ C_{2k}^+ / Q_r \leq R_{jk,BOD}^q, \forall k, j = 3 \quad (11e)$$

$$1.3767 + 0.8035(1 - \eta_{2k}^+) X_{2kk}^+ w_{2kk}^+ C_{2k}^+ / Q_r + (1 - \eta_{3k}^+) X_{3kk}^+ w_{3kk}^+ C_{3k}^+ / Q_r \leq R_{jk,BOD}^q, \forall k, j = 4 \quad (11f)$$

$$1.215 + 0.7091(1 - \eta_{2k}^+) X_{2kk}^+ w_{2kk}^+ C_{2k}^+ / Q_r + 0.8825(1 - \eta_{3k}^+) X_{3kk}^+ w_{3kk}^+ C_{3k}^+ / Q_r \leq R_{jk,BOD}^q, \forall k, j = 5 \quad (11g)$$

The main stream:

$$1.5576 + (1 - \eta_{1k}^+) X_{1kk}^+ w_{1kk}^+ C_{1k}^+ / Q_r \leq R_{jk,BOD}^q, \forall k, j = 1 \quad (11h)$$

$$1.2516 + 0.8035(1 - \eta_{1k}^+) X_{1kk}^+ w_{1kk}^+ C_{1k}^+ / Q_r + \left(1.215 + 0.7091(1 - \eta_{2k}^+) X_{2kk}^+ w_{2kk}^+ C_{2k}^+ / Q_r + 0.8825(1 - \eta_{3k}^+) X_{3kk}^+ w_{3kk}^+ C_{3k}^+ / Q_r \right) \leq R_{jk,BOD}^q, \forall k, j = 2 \quad (11i)$$

$$1.1045 + 0.7091(1 - \eta_{1k}^+) X_{1kk}^+ w_{1kk}^+ C_{1k}^+ / Q_r + 0.8825 \left(1.215 + 0.7091(1 - \eta_{2k}^+) X_{2kk}^+ w_{2kk}^+ C_{2k}^+ / Q_r + 0.8825(1 - \eta_{3k}^+) X_{3kk}^+ w_{3kk}^+ C_{3k}^+ / Q_r \right) + (1 - \eta_{4k}^+) X_{4kk}^+ w_{4kk}^+ C_{4k}^+ / Q_r \leq R_{jk,BOD}^q, \forall k, j = 6 \quad (11j)$$

$$0.9747 + 0.6258(1 - \eta_{1k}^{\pm}) X_{1kk}^{\pm} w_{1kk}^{\pm} C_{1k}^{\pm} / Q_r$$

$$+ 0.7788 \left(\begin{aligned} &1.215 + 0.7091(1 - \eta_{2k}^{\pm}) X_{2kk}^{\pm} w_{2kk}^{\pm} C_{2k}^{\pm} / Q_r \\ &+ 0.8825(1 - \eta_{3k}^{\pm}) X_{3kk}^{\pm} w_{3kk}^{\pm} C_{3k}^{\pm} / Q_r \end{aligned} \right) \quad (11k)$$

$$+ 0.8825(1 - \eta_{4k}^{\pm}) X_{4kk}^{\pm} w_{4kk}^{\pm} C_{4k}^{\pm} / Q_r$$

$$+ (1 - \eta_{5k}^{\pm}) X_{5kk}^{\pm} w_{5kk}^{\pm} C_{5k}^{\pm} / Q_r \leq R_{jk}^q \text{ BOD}, \forall k, j = 7$$

$$0.7358 + 0.4724(1 - \eta_{1k}^{\pm}) X_{1kk}^{\pm} w_{1kk}^{\pm} C_{1k}^{\pm} / Q_r$$

$$+ 0.5879 \left(\begin{aligned} &1.215 + 0.7091(1 - \eta_{2k}^{\pm}) X_{2kk}^{\pm} w_{2kk}^{\pm} C_{2k}^{\pm} / Q_r \\ &+ 0.8825(1 - \eta_{3k}^{\pm}) X_{3kk}^{\pm} w_{3kk}^{\pm} C_{3k}^{\pm} / Q_r \end{aligned} \right) \quad (11l)$$

$$+ 0.6661(1 - \eta_{4k}^{\pm}) X_{4kk}^{\pm} w_{4kk}^{\pm} C_{4k}^{\pm} / Q_r$$

$$+ 0.7548(1 - \eta_{5k}^{\pm}) X_{5kk}^{\pm} w_{5kk}^{\pm} C_{5k}^{\pm} / Q_r$$

$$+ (1 - \eta_{6k}^{\pm}) X_{6kk}^{\pm} w_{6kk}^{\pm} C_{6k}^{\pm} / Q_r \leq R_{jk}^q \text{ BOD}, \forall k, j = 8$$

Maximum allowable DO-deficit discharge:

The tributary:

$$0.582 + 0.171(1 - \eta_{2k}^{\pm}) X_{2kk}^{\pm} w_{2kk}^{\pm} C_{2k}^{\pm} / Q_r \leq R_{jk}^q \text{ D}, \forall k, j = 4 \quad (11m)$$

$$0.648 + 0.234(1 - \eta_{2k}^{\pm}) X_{2kk}^{\pm} w_{2kk}^{\pm} C_{2k}^{\pm} / Q_r \quad (11n)$$

$$+ 0.109(1 - \eta_{3k}^{\pm}) X_{3kk}^{\pm} w_{3kk}^{\pm} C_{3k}^{\pm} / Q_r \leq R_{jk}^q \text{ D}, \forall k, j = 5$$

The main stream:

$$0.555 + 0.171(1 - \eta_{1k}^{\pm}) X_{1kk}^{\pm} w_{1kk}^{\pm} C_{1k}^{\pm} / Q_r \leq R_{jk}^q \text{ D}, \forall k, j = 2 \quad (11o)$$

$$0.611 + 0.234(1 - \eta_{1k}^{\pm}) X_{1kk}^{\pm} w_{1kk}^{\pm} C_{1k}^{\pm} / Q_r$$

$$+ 0.109 \left(\begin{aligned} &0.648 + 0.234(1 - \eta_{2k}^{\pm}) X_{2kk}^{\pm} w_{2kk}^{\pm} C_{2k}^{\pm} / Q_r \\ &+ 0.109(1 - \eta_{3k}^{\pm}) X_{3kk}^{\pm} w_{3kk}^{\pm} C_{3k}^{\pm} / Q_r \end{aligned} \right) \leq R_{jk}^q \text{ D}, \forall k, j = 6 \quad (11p)$$

$$0.6435 + 0.277(1 - \eta_{1k}^{\pm}) X_{1kk}^{\pm} w_{1kk}^{\pm} C_{1k}^{\pm} / Q_r$$

$$+ 0.296 \left(\begin{aligned} &0.648 + 0.234(1 - \eta_{2k}^{\pm}) X_{2kk}^{\pm} w_{2kk}^{\pm} C_{2k}^{\pm} / Q_r \\ &+ 0.109(1 - \eta_{3k}^{\pm}) X_{3kk}^{\pm} w_{3kk}^{\pm} C_{3k}^{\pm} / Q_r \end{aligned} \right) \quad (11q)$$

$$+ 0.109(1 - \eta_{4k}^{\pm}) X_{4kk}^{\pm} w_{4kk}^{\pm} C_{4k}^{\pm} / Q_r \leq R_{jk}^q \text{ D}, \forall k, j = 7$$

$$0.6541 + 0.3242(1 - \eta_{1k}^{\pm}) X_{1kk}^{\pm} w_{1kk}^{\pm} C_{1k}^{\pm} / Q_r$$

$$+ 0.369 \left(0.648 + 0.234(1 - \eta_{2k}^{\pm}) X_{2kk}^{\pm} w_{2kk}^{\pm} C_{2k}^{\pm} / Q_r \right.$$

$$\left. + 0.109(1 - \eta_{3k}^{\pm}) X_{3kk}^{\pm} w_{3kk}^{\pm} C_{3k}^{\pm} / Q_r \right) \quad (11r)$$

$$+ 0.390(1 - \eta_{4k}^{\pm}) X_{4kk}^{\pm} w_{4kk}^{\pm} C_{4k}^{\pm} / Q_r$$

$$+ 0.205(1 - \eta_{5k}^{\pm}) X_{5kk}^{\pm} w_{5kk}^{\pm} C_{5k}^{\pm} / Q_r \leq R_{jkD}^q, \forall k, j = 8$$

$$0.6303 + 0.3288(1 - \eta_{1k}^{\pm}) X_{1kk}^{\pm} w_{1kk}^{\pm} C_{1k}^{\pm} / Q_r$$

$$+ 0.3817 \left(0.648 + 0.234(1 - \eta_{2k}^{\pm}) X_{2kk}^{\pm} w_{2kk}^{\pm} C_{2k}^{\pm} / Q_r \right.$$

$$\left. + 0.109(1 - \eta_{3k}^{\pm}) X_{3kk}^{\pm} w_{3kk}^{\pm} C_{3k}^{\pm} / Q_r \right) \quad (11s)$$

$$+ 0.411(1 - \eta_{4k}^{\pm}) X_{4kk}^{\pm} w_{4kk}^{\pm} C_{4k}^{\pm} / Q_r$$

$$+ 0.424(1 - \eta_{5k}^{\pm}) X_{5kk}^{\pm} w_{5kk}^{\pm} C_{5k}^{\pm} / Q_r$$

$$+ 0.152(1 - \eta_{6k}^{\pm}) X_{6kk}^{\pm} w_{6kk}^{\pm} C_{6k}^{\pm} / Q_r \leq R_{jkD}^q, \forall k, j = 9$$

(3) Product demand constraints:

$$T_{ikmin} \leq T_{ik}^{\pm} \leq T_{ikmax}, \forall i, k \quad (11t)$$

(4) Non-negative constraints:

$$X_{ikk}^{\pm} \geq 0, \forall i, k, h \quad (11u)$$

where j is index of reach ($j=1,2,\dots,9$), where $j=1$ for the upstream end, $j=3$ for tributary end and $j=9$ for the downstream end; i denotes the index of wastewater discharge source, where $i=1$ for the municipal wastewater-treatment plant, $i=2, 3, 4, 5$ for the industrial sectors (a paper mill, a chemical plant, a tannery plant, and a tobacco factory), and $i=6$ for the recreational sector; wastewater from these sources would enter the stream at the beginnings of reaches Y_2, Y_4-Y_5 and Y_7-Y_9 ; f^{\pm} is the net system benefit over the planning horizon (\$); q_{ik} denotes probability of random wastewater discharge rate at source i in period k with level h (%); h denotes the level of wastewater discharge rate at each source; k is the index of planning period; NB_{ik}^{\pm} is the net benefit per unit product from source i (\$/unit product); C_{ik}^{\pm} is the BOD concentration of raw wastewater generated at source i in period k (kg/m³); c_{ik}^{\pm} and d_{ik}^{\pm} are the coefficients of treatment cost functions for source i during period k ($c_{ik}^{\pm}, d_{ik}^{\pm} > 0$); Q_r is the stream flow

($10^3 \text{ m}^3/\text{day}$), where Q_{ik} is the amount of discharged wastewater from source i during period k ($10^3 \text{ m}^3/\text{day}$); η_{ik}^{\pm} is the BOD treatment efficiency at source i during period k (%); $R_{jk,BOD}^{\pm}$ is the designated BOD concentration at the beginning of reach j (mg/L); $R_{jk,D}^{\pm}$ is the allowable DO deficit at the end of reach j (mg/L); S_{ik}^{\pm} is the BOD discharge allowance for source i during period k (tonne/day); $T_{ik,\min}$ and $T_{ik,\max}$ are minimum and maximum demands for product i during period k (unit/day), respectively; T_{ik}^{\pm} is the product target pre-regulated by source i during period k (the first-stage decision variable) (unit/day); w_{ijk}^{\pm} is the random wastewater discharge rate at source i in period k with level h ($\text{m}^3/\text{unit product}$); X_{ijk}^{\pm} means the production level of source i during period k with level h , which is affected by the random BOD generation rates and the environmental requirements (the second-stage decision variable) (unit/day).

Model (11) can be solved according to the solution algorithm as described in Section 2. The objective is to maximize the expected net benefit subject to the environmental requirements under uncertainty over the planning horizon. The decision variables (X_{ijk}^{\pm}) are the planned production levels of different plants during different period. The constraints include the BOD discharge constraints, the maximum BOD discharge and DO-deficit constraints in the tributary and the main stream, the product demand constraints and non-negative constraints. The detailed solution process for the SITCQP model is summarized in Fig. 3. Fig. 3 describes different optimized methods, such as the quadratic programming, the two-stage stochastic programming and the interval-parameter programming, as well as the process of establishing the SITCQP model. The water quality model and the fitting cumulative distribution function have been imported in Fig. 3, which are used to describe the constraints of water quality. Fig. 3 also introduces the solving process of the developed model. The MATLAB[®] is used to establish the cumulative distribution functions of BOD discharge, and the LINGO[®] is employed to implement and solve the developed SITCQP model. A model debugging is also coded and utilized to determine whether the model can be solved. After the completion of the debugging, each run of the model can obtain a series of optimal solutions. The model can be solved with a high speed because of less decision variables.

Results and Discussion

By combining simulation and optimization, the production planning of each plant can be obtained at different levels of risk. The results indicate that any change in q_i will yield different water quality requirements and thus lead to different wastewater-discharge patterns. The solutions for decision variables (X_{ikk}^{\pm}) are presented as combinations of intervals. The upper bounds of X_{ikk}^{\pm} (i.e. X_{ikk}^{+}) correspond to a higher system benefit under advantageous conditions, while the lower bounds (i.e. X_{ikk}^{-}) are related to a more conservative strategy. Moreover, probabilistic deficit or surplus will occur if the pre-regulated targets are different from the planned levels, where probabilistic deficit/surplus = planned level-pre-regulated target (i.e.

$PD_{ikk}^{\pm} / PS_{ikk\ opt}^{\pm} = X_{ikk\ opt}^{\pm} - (T_{ik\ opt} + \Delta T_{ik} y_{ik\ opt})$). They indicate that a probabilistic deficit (with negative sign) may exist when the pre-regulated target exceeds the planned level; conversely, a surplus (with positive sign) will occur when the target is lower than the planned level. The solutions obtained through the SITCQP model under different q_i levels is given in Appendix A.

An analysis of the modeling solutions for period 1 under $q_i = 0.01$ are provided below, while those for periods 2, 3 and for $q_i = 0.05, 0.1$ can be similarly interpreted based on the results presented in Appendix A. The result of $y_{11\ opt} = 0.80$ indicates that the optimized water-supply target is 70,000 m³/day for the municipal sector in period 1 under $q_i = 0.01$. However, the planned levels would be [43,195, 77,504] m³/day if the wastewater discharge rate is low with a probability of 20%, [40,691, 73,871] m³/day under medium discharge rate with a probability of 60%, and [37,941, 69,526] m³/day under high discharge rate with a probability of 20%. Correspondingly, the probabilistic deficit (or surplus) levels will be [-26,804, 7504], [-29308, 3,871] and [-32058, -473] m³/day under low, medium and high discharge rates, respectively. Fig. 4 presents the optimized water supply to the municipal sector in different levels under $q_i = 0.01$. The tendency of the planned levels is significantly decreasing when the wastewater loading is increasing with time, while the production level appears a minor increase during different periods. Furthermore, the negative values represent the probabilistic deficits which would lead to exceeding wastewater discharge and consequently economic penalty due to the violation of

environmental requirements. This implies that, if the actual human activities are planned based on the regulating targets, the water deficits (subject to penalties) will approximately be 26,804, 29308 and 32058 m³/day when the wastewater discharge rates are low, medium and high, respectively. The penalties are presented in terms of the raised treatment costs and/or the punishments due to excessed wastewater discharges. In contrast, the positive values denote the probabilistic surpluses. They indicate that, under advantageous conditions, low wastewater discharge levels and high opportunity losses will possibly occur due to a significantly conservative strategy for economic activities.

Fig. 5 provides the optimized production for the industry over the planning horizon in different levels under $q_i = 0.01$. During period 1, the results of $y_{21opt} = 0$, $y_{31opt} = 0.73$, $y_{41opt} = 0.60$ and $y_{51opt} = 0.63$ indicate that the production targets will be 7.0, 41.0, 8.0 and 3.5 tonne/day for the paper mill, the chemical plant, the tannery plant and the tobacco factory, respectively. The solution of $y_{21opt} = 0$ indicates that the optimized target for the paper mill corresponds to its lower bound, implying a conservative attitude. For the paper mill, the planned production levels would be [6.29, 8.01], [5.99, 11.26] and [5.67, 10.28] tonne/day when the wastewater discharge rates are low, medium and high; correspondingly, the probabilistic deficit (or surplus) will be [-0.71, 1.01], [-1.01, 4.26] and [-1.33, 3.28] tonne/day, respectively. For the chemical plant, the planned level would be [35.25, 55.27], [32.32, 52.14], [30.29, 42.18] and [24.29, 40.26] tonne/day when the wastewater discharge rates are low, low-medium, medium and high; correspondingly, the probabilistic deficit (or surplus) will be [-5.75, 14.27], [-8.68, 11.14], [-10.71, 1.18] and [-16.71, -0.74] tonne/day, respectively. For the tannery plant, the planned level would be [5.80, 9.75], [9.29, 14.96], [5.27, 8.70] and [5.01, 8.42] tonne/day when the wastewater discharge rates are low, low-medium, medium and high; correspondingly, the probabilistic deficit (or surplus) will be [-2.20, 1.75], [1.29, 6.96], [-2.73, 0.70] and [-2.99, 0.42] tonne/day, respectively. For the tobacco factory, the planned level will be [1.99, 4.86], [1.89, 4.63], [1.80, 4.41] and [1.72, 4.20] tonne/day when the wastewater-discharge rates are low, low-medium, medium and high; correspondingly, the probabilistic deficit (or surplus) will be [-1.51, 1.36], [-1.61, 1.13], [-1.70, 0.91] and [-1.78, 0.70] tonne/day, respectively.

Fig. 6 shows the optimized area for the recreational sector in different levels under $q_i = 0.01$. The result of $\gamma_{61opt} = 0.63$ indicates that the optimized target will be $3.5 \text{ hm}^2/\text{yr}$ in period 1 under $q_i = 0.01$. In comparison, the planned level will be $[2.59, 3.56] \text{ hm}^2/\text{yr}$ if the wastewater-discharge rate is low with a probability of 10%, $[3.11, 4.87] \text{ hm}^2/\text{yr}$ under low-medium discharge rate (probability = 20%), $[2.76, 2.94] \text{ hm}^2/\text{yr}$ under medium discharge rate (probability = 40%), $[4.35, 6.70] \text{ hm}^2/\text{yr}$ under medium-high discharge rate (probability = 20%), and $[1.98, 2.43] \text{ hm}^2/\text{yr}$ under high discharge rate (probability = 10%). Correspondingly, the probabilistic deficit (or surplus) will be $[-0.91, 0.06]$, $[-0.39, 1.37]$, $[-0.74, -0.56]$, $[0.85, 3.20]$ and $[-1.52, -1.07] \text{ hm}^2/\text{yr}$, respectively.

Fig. 7 shows the solutions of the production alternatives for the industries under $q_i = 0.01, 0.05$ and 0.1 . It displays the relationship among q_i and the planned levels. Take the paper mill as an example, in period 1, under a significance level of $q_i = 0.01$, the planned production levels will be $[6.29, 8.01]$, $[5.99, 11.26]$ and $[5.67, 10.28]$ tonne/day when the wastewater-discharge rates are low, medium and high; similarly, under a significance level of $q_i = 0.05$, the planned production levels will be $[7.37, 8.56]$, $[8.15, 14.70]$ and $[7.69, 12.63]$ tonne/day when the wastewater-discharge rates are low, medium and high; finally, under a significance level of $q_i = 0.1$, the planned production levels will be $[17.40, 29.80]$, $[16.57, 28.37]$ and $[15.63, 27.02]$ tonne/day when the wastewater-discharge rates are low, medium and high. The solutions indicate that the planned production levels increase with the increased q_i levels. An increased q_i level means a relaxed allowable BOD discharge and DO-deficit constraints and thus a raised risk when violation occurs. Similar indications can be summarized for the chemical plant and the tannery plant (Fig. 7). However, it is observed that the solutions of the planned levels for the municipal sector ($i = 1$) and the tobacco factory ($i = 5$) have undetectable changes under different q_i levels. This is possibly caused by the reason that the chance constraints are no restrictive for the planned levels of the two sectors.

The solutions of the planning area for the recreational sector under $q_i = 0.01, 0.05$ and 0.1 are presented in Fig. 8. In period 1, under a significance level of $q_i = 0.01$, the planned level will be $[2.59, 3.56]$, $[3.11, 4.87]$, $[2.76, 2.94]$, $[4.35, 6.70]$ and $[1.98, 2.43] \text{ hm}^2/\text{yr}$ when the wastewater-discharge rates are low, low-medium, medium, medium-high and high. Similarly,

under a significance level of $q_i = 0.05$, the planned level will be [3.07, 3.98], [4.84, 6.01], [2.53, 3.28], [5.77, 7.64] and [2.09, 2.71] hm^2/yr when the wastewater-discharge rates are low, low-medium, medium, medium-high and high. Under a significance level of $q_i = 0.1$, the planned level will be [4.36, 5.02], [6.56, 8.30], [3.60, 4.15], [7.20, 9.53] and [2.98, 3.43] hm^2/yr when the wastewater-discharge rates are low, low-medium, medium, medium-high and high. Similar situations appear in periods 2 and 3.

Fig. 9 provides the expected net system benefit over the planning horizon under the different q_i levels. The solutions of f^\pm under $q_i = 0.01, 0.05$ and 0.1 are $[1064.1, 2505.6] \times 10^6$, $[1182.0, 2565.3] \times 10^6$ and $[1215.6, 2667.0] \times 10^6$ dollars, respectively. This indicates that, as the decision variables vary within their two bounds, the expected net system benefit will correspondingly change between f_{opt}^+ and f_{opt}^- , representing the optimistic and conservative strategies.

Decisions at a lower q level will possibly lead to an increased reliability in fulfilling the system requirements but with a higher cost; in contrast, decisions at a higher q level will possibly result in a lower cost, but the risk of violating the constraints may raise. Moreover, the relationship between q_i and f^\pm also demonstrates a tradeoff between the economic efficiency and the constraints violation risk due to multiple uncertainties that exist in various system components.

In the previous analyses, the optimization of the entire system has been fully taken into consideration. In the case of meeting water environment capacity of the stream, the maximum production of the plants along the stream can be obtained at a certain level of risk by combining simulation and optimization models. Take $q_i = 0.01, k = 1$ as an example, the maximum production of the plants of the tributary (i.e. the paper mill and the chemical plant) are [7.13, 28.57] tonne/day and [38.15, 61.37] tonne/day. Obviously, these values are higher than the calculated results above. Therefore, when the pollution loading of the stream system is low, the operators of some plants can appropriately increase their production, which will increase the economic benefits in the region. Moreover, this simulation model can provide an effective link between the wastewater treatment cost and the water quality goals. When the stream is heavily polluted, decision makers can identify and control the pollution sources by using simulation, which could not be done by traditional optimization methods. Therefore, the water quality of the

stream can be dynamically monitored and strictly controlled to meet the planning targets. The sensitivity analysis indicates that the chemical plant has the highest contribution to the pollution among all sectors due to its production scale and allowable BOD loadings. Therefore, the chemical plant should preferentially reduce its wastewater discharge and enhance its wastewater treatment efficiencies.

Generally, the above results demonstrate that the optimization model can effectively reflect the interval-format uncertainties in the optimization process, and generate inexact solutions that contain a spectrum of potential wastewater treatment options. The decision alternatives can be obtained by adjusting different combinations of the decision variables within their optimal intervals. The allowable levels of environmental violations could be determined based on discussions among stakeholders according to specific system conditions (e.g., system cost, system-failure risk, and river pollution situations). The local managers can make a decision with the guidance of the solutions (including the production capacity, the wastewater discharge rate, production trends and so on); meanwhile, under different conditions of the wastewater discharge rates, they can dynamically adjust their production schemes. Moreover, the SITCQP model can provide an effective link between environmental requirements and economic implications expressed as penalties or opportunity losses caused by improper policies. The results can help decision makers to manage stream water quality and gain insight into the tradeoffs between the environmental quality and treatment cost.

Conclusion

In this study, a simulation-based inexact two-stage chance constraint quadratic programming (SITCQP) method was developed for supporting decision making in stream water quality management under uncertainty. This method is a hybrid of the two-stage stochastic programming (TSP), the chance constraint programming (CCP), the inexact quadratic programming (IQP) and the multi-segment stream water quality simulation model. A multi-segment stream water quality simulation model based on the O'Connor model was provided for reflecting the relationship between the pollution-control actions before wastewater discharge and the environmental responses after the discharge. The interval quadratic polynomials were employed to reflect the

uncertainties and nonlinearities associated with the costs for wastewater treatment. The developed SITCQP method has advantages in reflecting uncertain, nonlinear, and dynamic features that exist in many system components, as well as the interactions in these features. In detail, the SITCQP model can handle uncertainties expressed as probability distributions and intervals, and deal with nonlinearities in the objective function.

The developed method was tested by a hypothetical case of water quality management in a stream with one tributary. The dynamic interactions between pollutant loading and water quality were reflected in the case study. The solutions are presented as combinations of interval and distributional information, and can thus facilitate communications for different forms of uncertainties and nonlinearities. The results are valuable for supporting local decision makers in generating cost-effective water quality management schemes. The results also indicate that the chemical plant has significant contribution to the stream pollution, which should be the prior plant to reduce the wastewater discharge and enhance the wastewater treatment efficiency. Moreover, the solutions from the SITCQP model can also be used to demonstrate the tradeoffs between the overall wastewater treatment cost and the system-failure risk due to inherent uncertainties that exist in various water quality system components. Although this study is the first attempt for planning a water quality management system through the SITCQP approach, this novel approach is also applicable to many other environmental management problems to help managers make decisions under dual uncertainties.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:

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Appendix

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Table 1 Wastewater-discharge rates and the associated probabilities

Probability		Time period		
		$k = 1$	$k = 2$	$k = 3$
Wastewater-treatment plant (m^3/m^3 supply)				
$h = 1$ (low)	0.2	[0.61, 0.65]	[0.61, 0.65]	[0.61, 0.65]
$h = 2$ (medium)	0.6	[0.64, 0.69]	[0.64, 0.69]	[0.64, 0.69]
$h = 3$ (high)	0.2	[0.68, 0.74]	[0.68, 0.74]	[0.68, 0.74]
Paper mill (m^3/tonne)				
$h = 1$ (low)	0.2	[269.1, 283.6]	[249.1, 262.6]	[229.1, 241.6]
$h = 2$ (medium)	0.6	[282.6, 297.8]	[261.6, 275.7]	[240.6, 253.7]
$h = 3$ (high)	0.2	[296.8, 315.7]	[274.7, 292.2]	[252.7, 268.9]
Chemical plant (m^3/tonne)				
$h = 1$ (low)	0.15	[107.6, 115.6]	[103.6, 111.6]	[95.5, 103.5]
$h = 2$ (low-medium)	0.25	[122.53, 130.53]	[119.52, 127.52]	[115.8, 123.8]
$h = 3$ (medium)	0.45	[127.35, 135.35]	[121.45, 129.45]	[112.4, 120.4]
$h = 4$ (high)	0.15	[135.3, 143.3]	[128.5, 136.5]	[119.6, 127.6]
Tannery plant (m^3/tonne)				
$h = 1$ (low)	0.15	[107.9, 114.3]	[101.8, 105.8]	[91.7, 97.3]
$h = 2$ (low-medium)	0.25	[113.3, 120.0]	[106.8, 112.2]	[96.3, 102.2]
$h = 3$ (medium)	0.45	[121.0, 126.0]	[111.2, 128.1]	[101.2, 107.3]
$h = 4$ (high)	0.15	[125.0, 132.3]	[127.1, 134.5]	[106.3, 112.7]
Tobacco factory (m^3/tonne)				
$h = 1$ (low)	0.15	[184.7, 194.9]	[184.7, 194.9]	[184.7, 194.9]
$h = 2$ (low-medium)	0.25	[193.8, 204.6]	[193.8, 204.6]	[193.8, 204.6]
$h = 3$ (medium)	0.45	[203.5, 214.8]	[203.5, 214.8]	[203.5, 214.8]
$h = 4$ (high)	0.15	[213.7, 225.5]	[213.7, 225.5]	[213.7, 225.5]
Recreation sector ($\text{m}^3/\text{hm}^2/\text{yr}$)				
$h = 1$ (low)	0.1	[2664.4, 2941.8]	[2421.3, 2674.4]	[2200.2, 2431.3]
$h = 2$ (low-medium)	0.2	[2931.8, 3236.0]	[2664.4, 2941.8]	[2421.3, 2674.4]
$h = 3$ (medium)	0.4	[3226.0, 3559.6]	[2931.8, 3236.0]	[2664.4, 2941.8]
$h = 4$ (medium-high)	0.2	[3549.6, 3915.6]	[3236.0, 3559.6]	[2931.8, 3236.0]
$h = 5$ (high)	0.1	[3905.6, 4307.1]	[3549.6, 3915.5]	[3226.0, 3559.6]

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Table 2 BOD concentrations and treatment efficiencies

	Wastewater- treatment plant	Paper mill	Chemical plant	Tannery plant	Tobacco factory	Recreational sector
Efficiency (%), η_i^\pm	[87, 91]	[82, 86]	[79, 82]	[79, 83]	[89, 94]	–
BOD concentration, C_k^\pm (kg/m ³)	[0.20, 0.22]	[0.31, 0.34]	[0.30, 0.32]	[1.1, 1.2]	[2.1, 2.3]	[0.05, 0.06]

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Table 3 Benefits and costs analysis for the sectors in the management system

	Time period		
	$k = 1$	$k = 2$	$k = 3$
Net benefits from different products, NE_k^{\pm}			
Water supply (\$/m ³)	[4.2, 5.1]	[4.6, 5.6]	[5.1, 6.2]
Paper (\$/tonne)	[366.3, 439.6]	[402.9, 483.6]	[443.2, 532.0]
Chemical product (\$/tonne)	[370.6, 380.6]	[421.2, 431.2]	[472.4, 482.4]
Tannery (\$/tonne)	[1104, 1536]	[1159.2, 1612.8]	[1182.4, 1645.1]
Tobacco (\$/tonne)	[11,000, 14,000]	[10,500, 13,500]	[10,000, 13,000]
Recreational sector (\$/hm ²)	[129.0, 161.2]	[135.4, 169.2]	[138.2, 172.6]
Cost for wastewater treatment, (10 ³ \$/yr)			
C_{1k}^- (Wastewater-treatment plant)	31.75+10.25 Q	33.65+10.86 Q	35.67+11.51 Q
C_{1k}^+ (Wastewater-treatment plant)	34.39+11.61 Q	36.45+12.31 Q	38.64+13.05 Q
C_{2k}^- (Paper mill)	33.52+7.18 Q	35.53+7.61 Q	37.66+8.07 Q
C_{2k}^+ (Paper mill)	35.56+8.71 Q	37.69+9.23 Q	39.64+5.46 Q
C_{3k}^- (Chemical plant)	32.38+4.86 Q	32.49+5.01 Q	33.28+5.51 Q
C_{3k}^+ (Chemical plant)	33.88+6.36 Q	33.59+6.52 Q	34.78+7.02 Q
C_{4k}^- (Tannery plant)	35.28+4.86 Q	37.40+5.15 Q	41.24+6.24 Q
C_{4k}^+ (Tannery plant)	36.71+5.56 Q	38.91+5.89 Q	38.64+13.05 Q
C_{5k}^- (Tobacco factory)	30.73+15.27 Q	32.57+16.19 Q	34.52+17.16 Q
C_{5k}^+ (Tobacco factory)	33.05+17.02 Q	35.03+18.04 Q	37.13+19.12 Q

Note: $Q = 10^3 \text{ m}^3/\text{day}$.

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Table 4 Allowable BOD loading for each wastewater source

	Allowable BOD loading, S_k^{\pm} (kg/day)		
	$k = 1$	$k = 2$	$k = 3$
Wastewater-treatment plant	[803, 851]	[883, 921]	[951, 1003]
Paper mill	[302, 348]	[299, 339]	[289, 321]
Chemical plant	[301, 345]	[297, 338]	[281, 325]
Tannery plant	[281, 317]	[248, 287]	[229, 251]
Tobacco factory	[98, 113]	[81, 101]	[75, 93]
Recreational sector	[113,515, 127,750]	[113,515, 127,750]	[113,515, 127,750]

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Table 5 Pre-regulated targets and market demands

	Time period		
	$k = 1$	$k = 2$	$k = 3$
Pre-regulated target			
Water supply (m^3/day)	[50,000, 75,000]	[55,000, 85,000]	[60,000, 90,000]
Paper (tonne/day)	[7, 15]	[8, 15]	[10, 18]
Chemical product (tonne/day)	[30, 45]	[30, 45]	[30, 45]
Tannery (tonne/day)	[5, 10]	[5, 10]	[5, 10]
Tobacco (tonne/day)	[1, 5]	[1, 5]	[1, 5]
Recreational activity (hm^2/yr)	[1, 5]	[1, 5]	[1, 5]
Minimum market demand			
Water supply (m^3/day)	60,000	65,000	70,000
Paper (tonne/day)	8	9	11
Chemical plant (tonne/day)	35	35	35
Tannery (tonne/day)	7	7	7
Tobacco (tonne/day)	2.5	2.2	2.0
Recreational activity (hm^2/yr)	1.5	2	2
Maximum market demand			
Water supply (m^3/day)	70,000	75,000	80,000
Paper (tonne/day)	10	11	13
Chemical plant (tonne/day)	40	40	40
Tannery (tonne/day)	9	9	9
Tobacco (tonne/day)	3.5	3.0	2.7
Recreational activity (hm^2/yr)	3	3.5	3.5

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Table 6 Water quality requirements for BOD and DO deficit in different q_i

	Different probability q_i		
	$q_i = 0.01$	$q_i = 0.05$	$q_i = 0.1$
$R_{BOD_{jk}}$ (mg/L)	5.5	6.5	7.5
$R_{D_{ik}}$ (mg/L)	2.5	3.5	4.5

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Fig. 8. Planning area for the recreational sector under different q_i levels

Fig. 9. Relationships between expected net system benefits and q_i levels

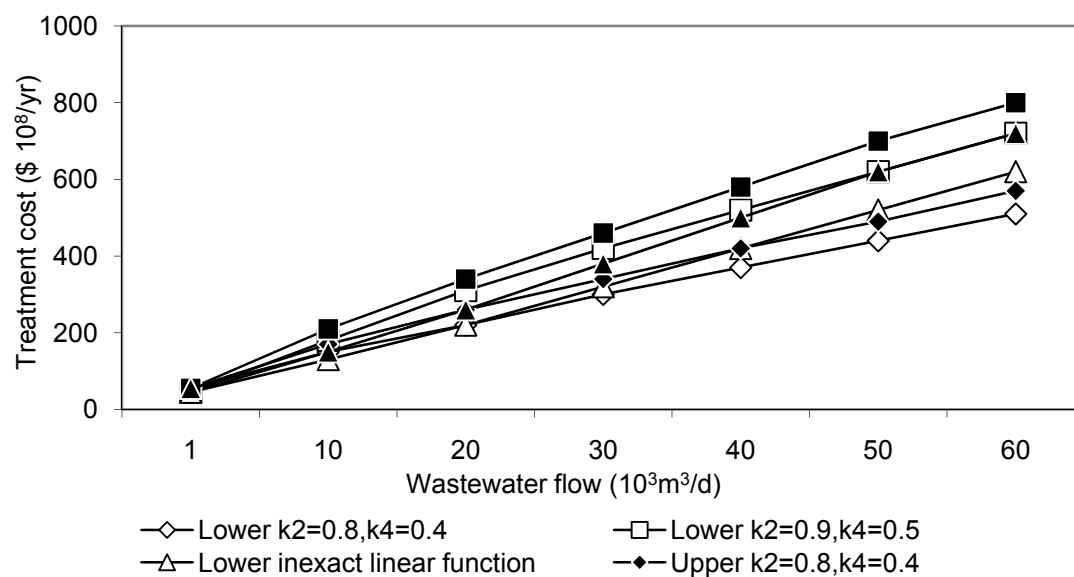


Fig. 1. Inexact nonlinear function between wastewater flow vs treatment cost

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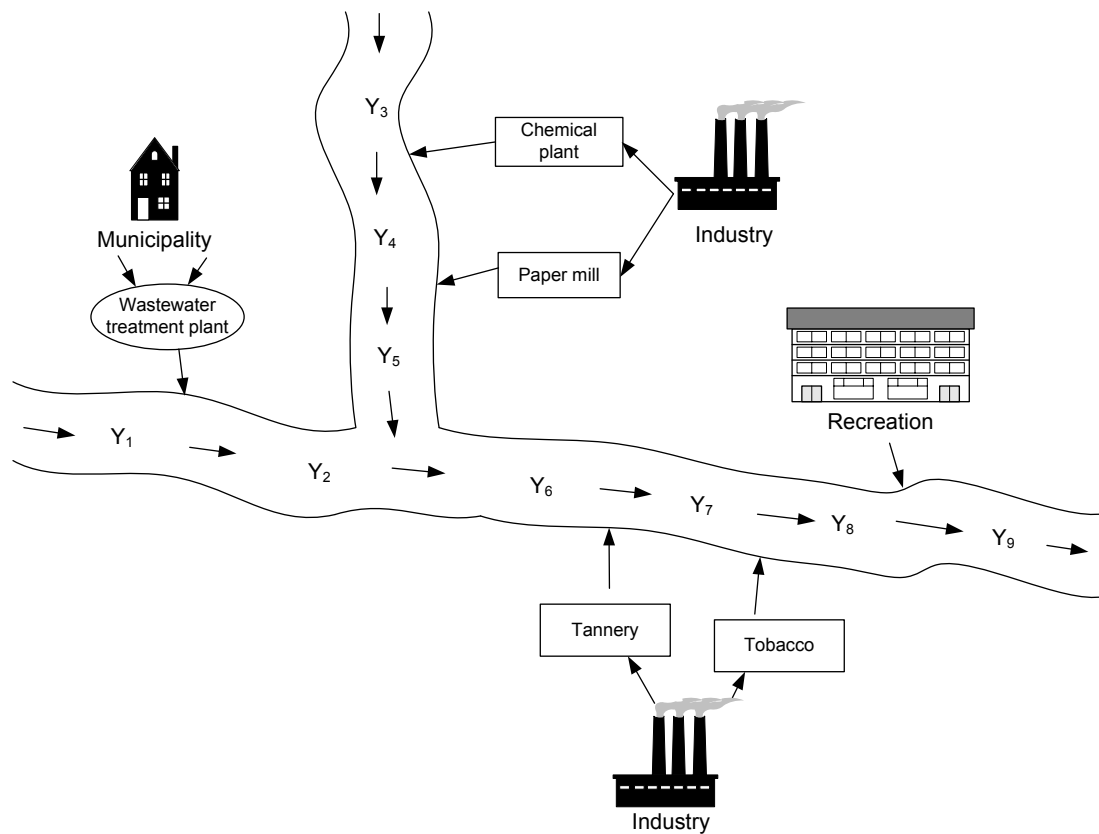


Fig. 2. Schematic diagram of the study system Note: Y_j is the length of reach j ($j=1, 2, \dots, 9$), where $Y_1=4\text{Km}$, $Y_2=3.5\text{ Km}$, $Y_3=4\text{Km}$, $Y_4=3.5\text{Km}$, $Y_5=2\text{Km}$, $Y_6=2\text{Km}$, $Y_7=2\text{Km}$, $Y_8=4.5\text{Km}$ and $Y_9=3\text{Km}$; L_0 is BOD concentration at the head of reach 1 and reach 3 ($L_0=2\text{mg/L}$); $k_a=0.62\text{ day}^{-1}$ and $k_d=0.5\text{ day}^{-1}$, where stream temperature is about $20\text{ }^\circ\text{C}$; Q is the stream flow ($Q=335,000\text{ m}^3/\text{day}$); u is the average flow velocity ($u=8.0\text{ km/day}$); D_0 is initial DO deficit of the main stream and the tributary ($D_0=0$).

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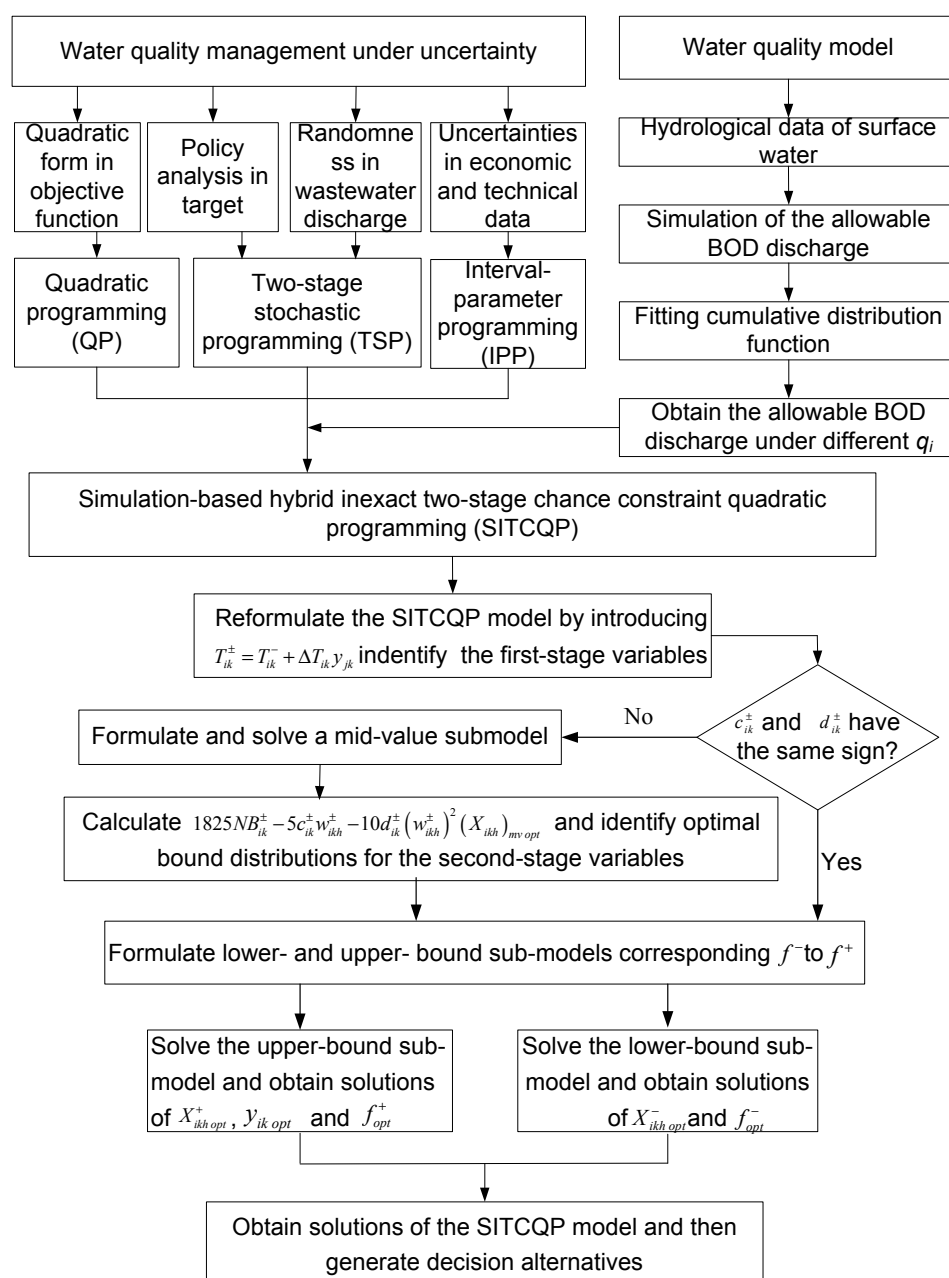


Fig. 3. Solution process for the SITCQP model

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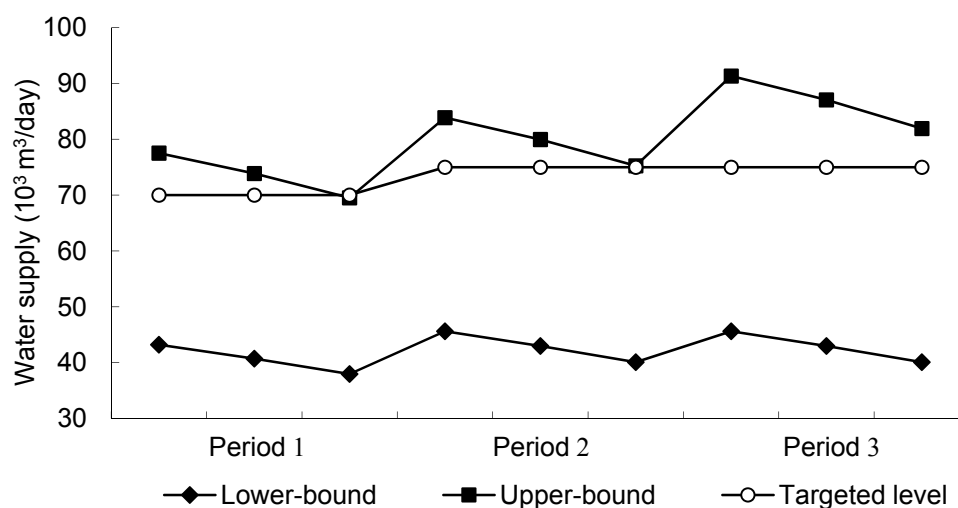


Fig. 4. Optimized water supply to the municipal sector in different levels under $q_i = 0.01$

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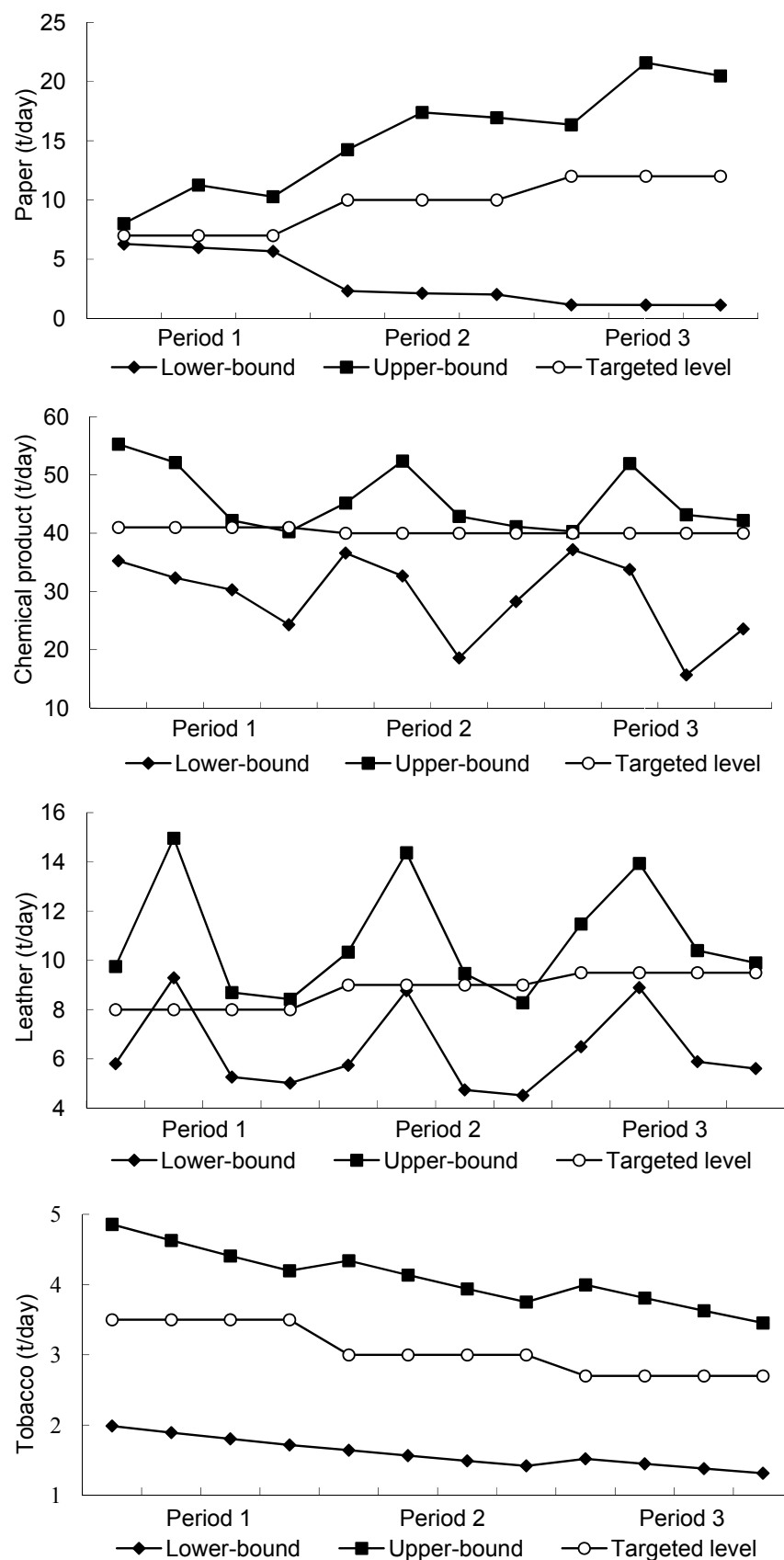


Fig. 5. Optimized production for the industry in different levels under $q_i = 0.01$

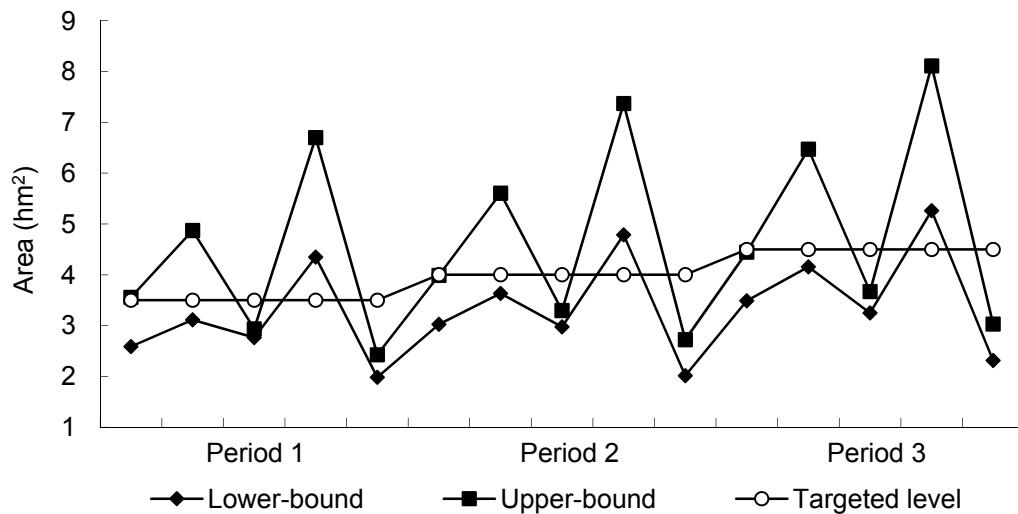


Fig. 6. Optimized area for the recreational sector in different levels under $q_i = 0.01$

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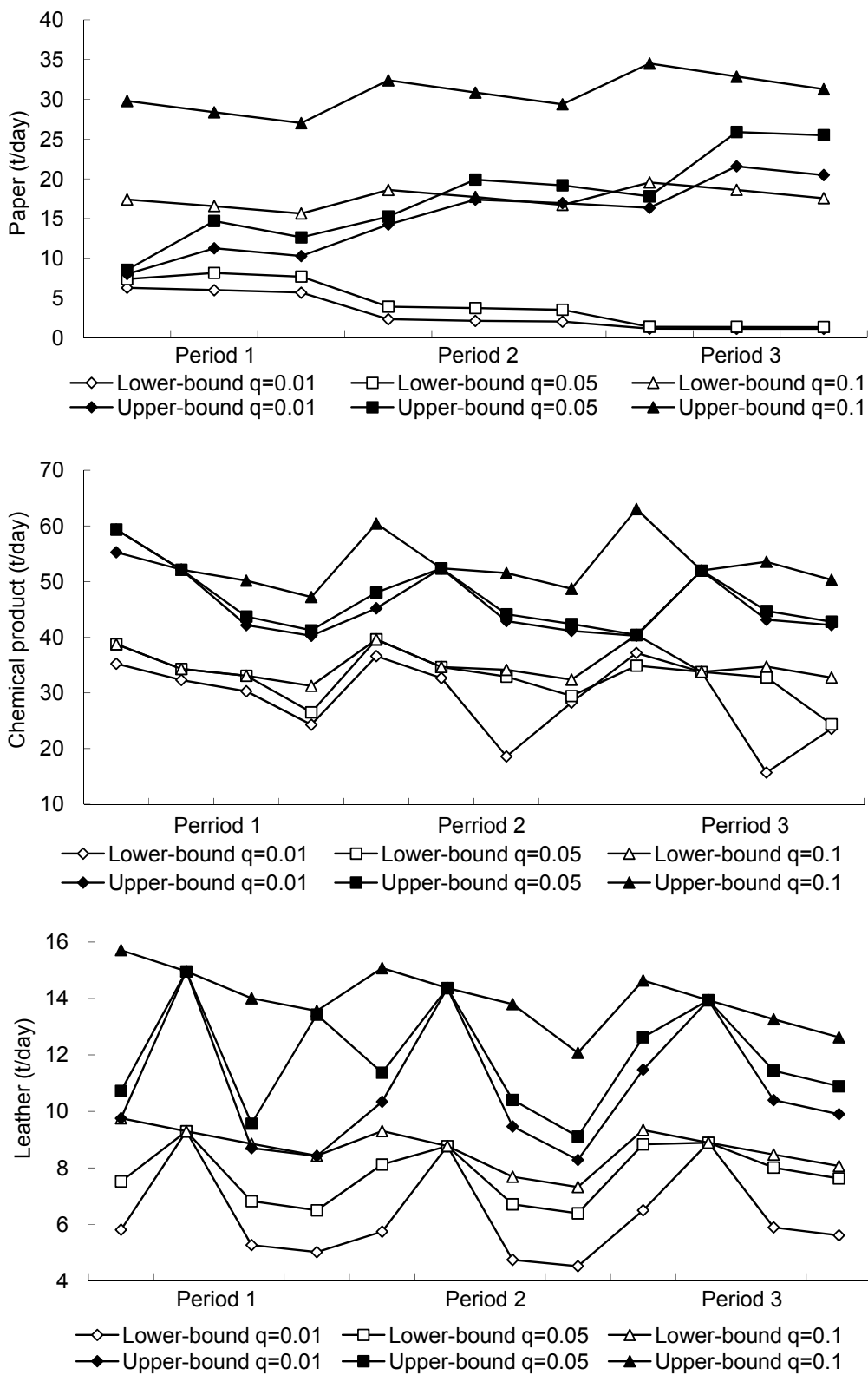


Fig. 7. Planning production for the industry under different q_i levels

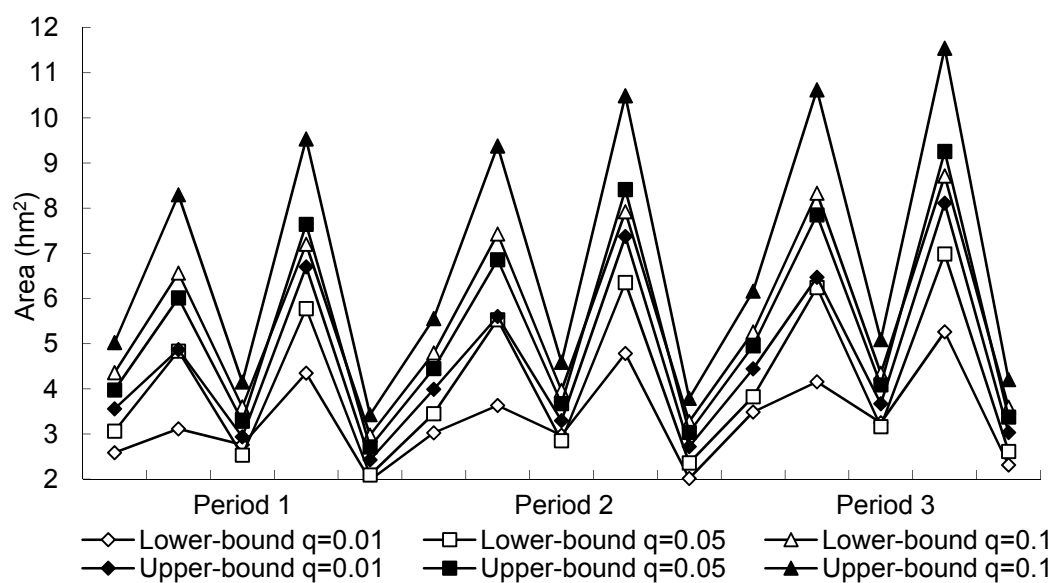


Fig. 8. Planning area for the recreational sector under different q_i levels

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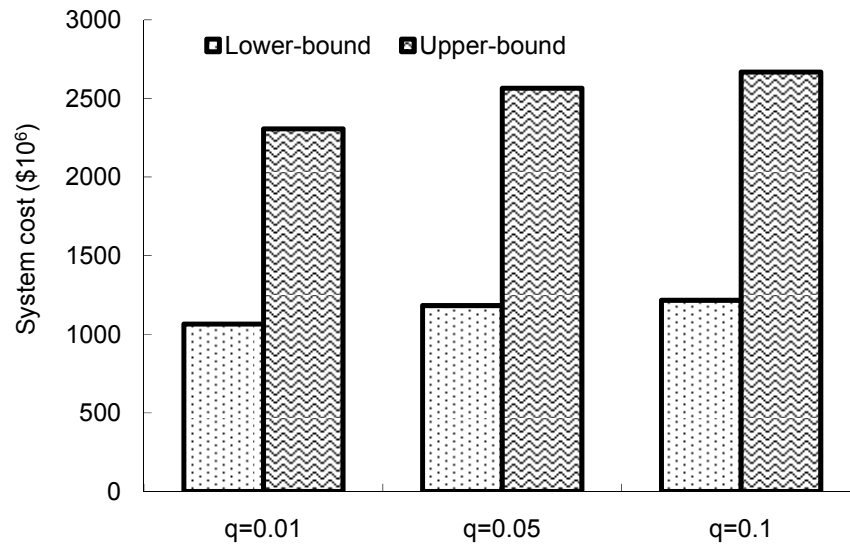


Fig. 9. Relationships between expected net system benefits and q_i levels

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